**Module 1 Tools of Finance**

Understanding the time value of money concept is key to developing a good grasp of how finance works. The time value of money means it is better to receive money sooner than later. The money received sooner can be invested to gain a return, which results in more money.

Cash flows that are received in the future have a lower value than cash flows in the present time. This is important for companies and it is necessary that managers can use the tools of finance to value these cash flows. Cash flows occurring at different points in time can be brought back to a current time value known as the present value. Bringing cash flows back to the current time allows cash flows to be compared and for more informed decisions to be made on company investments.

The basic set of tools that managers use are the techniques of present value and future value, using annuities to assist the valuations and expected return and standard deviation. The latter two tools gives an answer to the questions of what is the return an investment will make and what is the risk that is involved in producing that return.

It is essential that the present value techniques are learned and understood. This is necessary for a better understanding of the valuation processes for bonds, stocks and projects. Without a good grasp of these techniques you will find it difficult to gain a proper understanding of finance.

What you will learn here is how to value simple cash flows and how to use the statistical tables in the appendix. You will also learn where these techniques are used in everyday activities. It is important that you learn how to use the financial tables. Financial calculators will also help provide the answers to different problems, but they do not help build an intuitive feel for finance in the way the financial tables do. Financial calculators will deliver quick answers to complex problems, but for the finance course and for the exams a financial calculator is not necessary. A scientific calculator is enough to work through the problems in this text.

**Types of cash flows**

Single cash flow: This will be a single cash flow that is expected at some point in the future or is invested today for a number of periods, eg, £10,000 to be received at the end of five years or £5,000 invested today.

Annuity cash flow: This is a stream of equal cash flows over a period of time, eg, £2,000 per annum for five years. The period does not have to be a year, the financial tables are for any period of time; it could be years, or months or weeks or half-years. The cash flows in this example would be £2000 at the end of the first year, then £2000 at the end of year 2, £2000 at the end of year 3, £2000 at the end of year 4, and finally £2000 at the end of year 5.

Mixed cash flows: This is a stream of cash flows that are not equal, eg, £2000, £3000, £3500, £4000. This is not an annuity and cannot be valued using the annuity tables.

**Which table to use?**

There are four present value and future value tables in the appendix. There is the present value of a single cash flow (Table A1.1), present value of an annuity (Table A1.2), the future value of a single cash flow (Table A1.3) and the future value of an annuity (Table A1.4).

When using the tables the assumption is that the cash flow occurs at the end of the period. With a cash flow of £100 expected at the end of year 1, you would look up Table A1.1, one period and read across for the relevant interest rate and then down to obtain what is known as the present value interest factor (PVIFi,n).

For all four tables;

Table A1.1 Present value single cash flow PVIFi,n

Table A1.2 Present value annuity cash flow PVAIFi,v

Table A1.3 Future value single cash flow FVIFi,n

Table A1.4 Future value annuity cash flow FVAIFi,n

**Timeline**

A useful device to use to picture the cash flows you are trying to value is a timeline. This is a linear representation of the timings of the expected cash flows. This will show you what you are doing in the valuation exercise. This helps cement the concepts of present value and future value.

For example, you expect to receive £1000 in year 3 and £2000 in year 4. This would be represented on a timeline as follows;

Year 0 1 2 3 4

Today £1000 £2000

In the timeline, Year 0 represents the present time. Year 1 is the end of the first year and is one year later. Year 2 is the end of the second year and is two years from now.

Year 1 both represents the end of year one and the start of the second year. The timeline is useful for tracking the cash flows. On the timeline, cash inflows will be positive and cash outflows will be negative and will be signalled with a negative sign, eg, a payment by you of £4000 would be represented as -£4000.

Timelines are very useful when you are first learning how to understand the present value and future value techniques, as it might seem a bit abstract moving cash flows backwards and forwards in time. But timelines should be something you keep using as you progress through finance. Cash flows will become more complex and using the timeline is a good way of simplifying the problem and representing it as a picture.

**Present value of a single cash flow**

There are two ways that we will express the present value of a single cash flow. If we take the example of a company expecting to receive £20,000 in one year’s time, the present value can be represented as;

PV = £20,000  
 (1+i)t

or, using the tables;

PV = £10,000 \* PVIFi,n

If the interest rate is 10%, the calculations are;

PV = £20,000  
 (1.10)1

PV = £18,181.82

or using the tables;

PV = £20,000 \* PVIF0.10,1

PV = £20,000 \* 0.9091

PV = £18,182

The present value of £50,000 to be received in eight years’ time is;

PV = £50,000  
 (1.10)8

or

PV = £50,000 \* PVIF0.10,8

PV = £50,000 \* 0.4665

PV = £23,325

The format that has been used for this calculation using the tables is;

PV = FV \* PVIFi,n

This is the basic format that is used for all the tables, adjusting for the type of cash flow you are calculating. For example, the present value of annuity format would be;

PVA = FV \* PVAIFi,n

**Using the tables to find the interest rate**

The tables will not just work out the present value of a single cash flow, but if you are given the two cash flows (present value and future value) you can use the tables to work out the interest rate that holds for these two values.

Example:

An investment of £2000 returns £3525 after 5 years, what is the rate of interest (or return) that has been earned?

Using the tables and the present value expression;

PV = CF \* PVIFi,n

You need to identify the missing item. In this case you have the PV, the CF and the number of years, n, so the missing item is i, the interest rate.

Rearrange the expression to solve for the missing item;

PVIFi,n = PV  
 FV

In this case;

PVIFi,5 = £2000  
 £3525

PVIFi,5 = 0.5674

This figure is the present value interest factor for five years and the interest rate earned. To find the interest rate, look up 0.5674 in Table A1.1 and read across row 5 for this figure. The interest rate is 12%.

*Using a calculator to find the rate you would divide 3525 by 2000 giving 1.7625 and take the answer to the power of one divided by the number of years, ie = 1.1200. Subtract 1.0 to obtain the interest rate*

*Calculator: interest rate =*

Practice question

An investment of £60,000 will result in a cash flow of £95,208 in six years time. What rate of return has been earned? Use the tables to obtain your answer.

Solution

Use the PV expression;

PV = FV \* PVIFi,n

The missing item is the interest rate, so rearrange the expression to give;

PVIFi,n = PV  
 FV

PVIFi,6 = £60,000  
 £95,208

PVIFi,6 = 0.6302

Look up Table A1.1 and 6 years. Read along for 0.6302, the rate of return is 8%.

This present value interest factor (PVIFi,n) is also known as the discount factor. The rate of return or interest rate that has been calculated in these examples is also known as the discount rate.

**What is the importance of these discount rates?**

These discount rates are the opportunity cost of capital. This is the required rate of return on an investment. In terms of present value, the higher the discount rate applied to a future cash flow, the lower the present value will be. This is important for companies. Companies have what is known as a cost of capital. This is the discount rate that they apply to the cash flows of the projects they undertake. The higher the cost of capital (discount rate), the lower the present value of the cash flows of the project.

Another feature of the discount rate is that the discount factor is lower the further out in time you go. The discount factor for four years is lower than that of three years. What this means for companies is that cash flows are more valuable in present value terms the closer they are in time to now.

For companies evaluating their cash flows, timing, magnitude and risk are the key factors in valuing the cash flows. The closer in time they are, the more valuable they are, the larger the cash flows are the better for the firm, and the lower the risk, the more valuable they are.

This effect is shown in Figure 1.1 below;

The figure shows the present value of £100 given different discount rates and different time periods. The figure shows that as time lengthens, the present value falls, and as the discount rate rises, the present value falls. The message to companies is, to get their discount rate as low as they can and try and realise their cash flows as early as they can. How companies can lower their discount rate will be the subject of the Capital Structure Module.

Figure 1.1

The figure shows that if there was no discounting, the cash flows retain their original value. At 5% discount rate and 7 years the £100 has a present value of £71.07 (£100 \* PVIF0.05,7). At a 20% discount rate and 7 years, the present value will only be £27.91 (£100 \* PVIF0.20,7). If the cash flow was realised in year 4 instead of year 7 and the discount rate was 5%, then the PV is £82.27.

From the figure and the examples, it can be seen how sensitive cash flows are to changes in the discount rate and the number of years discounting. The present value of cash flows is inversely related to the discount rate and number of years. It is crucial for anyone working in business to be aware of the importance of the time value of money.

**Future value of a single cash flow**

When cash flows are invested, they earn interest and that interest is added to the original cash flow and then the interest will earn interest in the second period. This is known as compounding the interest. If there is no compounding, then the interest will be simple interest. You will only receive interest on your original cash flow. You will not receive interest on interest.

Bank accounts and savings products are based on compound interest as is mortgage and personal borrowing. Where would you think you would observe simple interest? It does not sound like a good deal when compared to compound interest. One of the biggest financial markets in the world is based on simple interest. This is the bond market. When you buy a bond you pay for example £100 and you will receive a fixed return of say 5%. This means that each year you hold the bond, you will receive £5 in interest payments (in bond terms these are known as coupon payments).

**Compound interest**

If you invest £100 and receive 6% per annum compounded from a bank, then after one year you will have £106 (£100 principal and £6 interest). Leaving that in the bank for another year you will have £112.36 (£106 \* 1.06). After three years, you will have £119.10 (£112.36 \* 1.06).

The formula for compound interest is; FV = CF \* (1+i)t. We can also express this using the financial tables. This is FV = CF \* FVIFi,n.

With the example above, how much will the original £100 have grown to if left in the bank until the end of the tenth year? Look up Table A1.3, read along row 10 and down the 6% column. The future value interest factor (FVIF0.06,10) is 1.7908. Multiply the cash flow by this to obtain £179.08.

The compound interest expression is also very important in finance because it can express the growth rate of different items, such as dividends, earnings per share, sales, cost of sales, capital expenditure. These are all important numbers in a company’s valuation and analysts will pay very close attention to these growth rates for evidence that a company is either slowing down or increasing the pace of growth, which will in turn affect the buy or sell recommendation they put on the stock.

**Compound growth of dividends an example.**

Giant-fone have just paid a dividend of 15 pence (£0.15). Four years ago the company paid a dividend of 11 pence. What is the growth rate of dividends?

This is just a compound growth problem. If it is framed in terms of the future value expression;

FV = PV \* FVIFi,n, we have, PV, FV, and n. It is only i that is missing.

Rearrange the expression to give

FVIFi,4 = FV  
 PV

FVIFi,4 = 15  
 11

FVIFi,4 = 1.3636

Looking up Table A1.3 and the row for four years, read across for 1.3636, then up for the growth rate. The closest number is 8% at 1.3605.

A calculator will give the exact growth rate here; i =

i =

i = 1.36360.25 – 1

i = 8.06%

The compound growth rate of dividends is just over 8%. This is an important figure as it is part of the share valuation model (that you will come on to in Module 2).

The compound growth expression is a very powerful tool in finance. The growth rate of different series (such as dividends, earnings, capital expenditure) is of great interest to many different people. To managers, who will see it as a tool for interpreting the success of their strategy; and to analysts who want to value the company for investors.

The expression is very simple, but is used in a large number of situations. The expression can be adjusted to find the number of years needed to achieve a certain return or the rate of return needed to achieve a certain target within a specified time frame.

Companies and investors wish to harness the power of compounding. Take two companies, they start off with earnings per share of 10 pence each and a share price of 100 pence each. One firm grows at 5% per annum and the other company grows at 15% per annum. After 10 years the earnings per share of the first company has reached 16.29 pence and it has reached 40.46 pence for the second company. Why is this important? One way of valuing companies is as a multiple of its earnings and the faster the growth the higher the multiple. This is the price earnings ratio, which will be discussed in more depth in Module 2. Company 1 is growing slowly at 5% per year, it may be priced at 10 times its earnings per share, giving a share price of 163 pence. Company 2 is growing at a faster rate, so will be valued at a higher multiple, maybe 15 times earnings. This would put its share price at 607 pence. Investors like companies that can grow quickly. This is what propelled Apple’s share price in the decade up to the start of 2013.

The figure below shows the power of compounding over time. £100 with no interest is worth £100 at the end of seven years. At 20% compound interest it is worth £358. If the interest was only simple interest, at 20% after 7 years you would have £240. After 20 years at 20% per annum compound you would have £3,834 (with simple interest you would have £500).

Figure 1.2

**Inter-period compounding**

Interest can be compounded more than once a year. If a bank charges 9% interest compounded once a year on a loan, then 9% is the actual rate of interest that will be paid. Here the interest is applied once a year, so the quoted interest rate is the actual rate that the borrower will pay. But if the interest is applied more than once a year, then the quoted rate will be lower than the rate actually paid by the borrower. With annual compounding the interest is applied at the end of the year, but if there was semi-annual compounding, then the interest would be applied twice; once after six months and again at the year end. So what interest rate does that work out to be? The interest rate that we need is the true rate that reflects how much the borrower will pay. This will be higher than the quoted rate because the amount borrowed has interest applied to it after six months and the interest added to the principal and the second six months interest will be on a higher amount of principal. The true rate is also known as the effective annual rate (EAR).

The following formula is used;

EAR = [Equation 1.1 ]

If a quoted annual interest rate is 8% and the interest is compounded quarterly, what is the effective annual interest rate?

Using equation 1. 1 , the effective annual interest rate can be found;

EAR =

EAR =

EAR = 1.08243 – 1

EAR = 0.08243 = 8.243%

The effective annual interest rate adjusts for the number of compounding periods per year to give a true interest rate. The quoted interest rate is just an annual rate , where the interest rate per period has been multiplied by the number of periods in a year (eg, from above the per period interest rate is 2% per quarter and the annual quoted rate is simply the 2% multiplied by 4 to give 8%). The interest rate per period is sometimes quoted (eg, 2% per quarter). Credit card companies often quote per period, but they must also report the effective annual interest rate.

If you are faced with interest rate quotes that are expressed in terms of different per period interest rates, you can now convert these into effective annual interest rates (EAR) and compare the rates for the most favourable rate for your situation.

**Continuous compounding**

Interest rates can be compounded at more frequent intervals. Daily compounding or even hourly compounding can be applied. Continuous compounding is when there is no gap between the compounding periods. The formula for continuous compounding is;

*eit -1*

where e is the exponential function, which has a value of 2.71828, and t is the number of years, and I is the interest rate. Applying this to the 8% annual interest rate from above we obtain the following continuously compounded interest rate;

*eit* = 2.718280.08\*1 - 1

*eit* = 1.083287 – 1

*eit* = 0.083287 = 8.3287%

**How much do you repay?**

The inter-period compounding formula can be slightly adjusted to give the formula for the amount that will be repaid or earned during a period where there has been more than annual compounding. In the formula, n stands for the number of years.

FV = PV \*

If we had borrowed £3,000 at 8% compounded quarterly for three years, how much would we have to repay at the end of three years?

FV = £3,000 \*

FV = £3,000 \*

FV = £3,000 \* 1.26824

FV = £3,804.72

You will repay £3,804.72 and your effective annual rate is 8.243%. You can see that the only difference between the formulas is that we no longer subtract 1 at the end. That would give the interest rate. Not subtracting the 1 gives a future value interest factor (FVIFi,n). We also can multiply the number of compounding periods, *m,* by the number of years, *n.*

**Finding the interest rate**

In the future value expression, we have FV, PV, i and n. If we have the future value (FV), the present value (PV), and the number of years, but not the interest rate, how is the interest rate calculated?

When using the tables the basic future value expression is written as;

FV = PV \* FVIFi,n

If the interest rate is missing you have to find FVIFi,n. To do this just rearrange the expression, solving for the missing item;

FVIFi,n = FV  
 PV

Worked example;

A cash flow today of £50,000 will grow into £70,925 by the end of 6 years. What is the interest rate that would achieve that outcome?

Rearranging the future value expression as above and solving for the missing interest rate gives;

FVIFi,n = £70,925  
 £50,000

FVIFi,n = 1.4185

Look up Table A1.3 and read along the 6 period row until you get to 1.4185, read up for the interest rate, which is 6%. (Solving on a calculator; divide FV/PV to get 1.4185. Take this to the power of 1 divided by the number of years; 1.4185(1/6) = 1.059997. Subtract 1 to obtain the interest rate = 6%.)

**How many years?**

Similar to the missing interest rate, you can work out the number of years needed to reach a certain target. For example, how long will it take you to double your investment of £25,000 if the interest rate is 9%?

The future value expression is;

FVIFi,n = FV  
 PV

The missing item is time and we are just rearranging for the missing item.

FVIFi,n = £50,000  
 £25,000

FVIFi,n = 2.0000

Look up Table A1.3 and read down the 9% column until you come to 2.0000. The closest answer is 1.9926, which is 8 years. So it will take approximately 8 years to double your investment if the interest rate is 9%.

**Multiple Cash Flows**

We have seen how you use the Finance Tables to value single cash flows, both present valuing and future valuing. We now look at multiple cash flows. There are two types of multiple cash flows; a stream of mixed (unequal) cash flows and a stream of equal cash flows.

The equal cash flows are annuity cash flows and can be valued quickly using the tables. The mixed cash flows have to be set out and valued individually.

**Annuity cash flows**

Annuity cash flows are a series of equal cash flows that occur over an evenly spaced time horizon, eg, you receive £2,000 per annum for four years. An ordinary annuity is an annuity where the cash flows occur at the end of each period, with this example; you would receive the £2,000 at the end of each of the next four years. An annuity due is an annuity where the cash flows occur at the beginning of the year, with our example, you would receive the £2,000 at the start of each year.

**How do you value these annuities?**

The Finance Tables are set out with the assumption that the cash flow occurs at the end of the period. The annuity tables A1.2 and A1.4 assume the annuities are ordinary annuities and the cash flows are at the end of the year.

In this example, if the interest rate was 7%, what would be the present value of the annuity? You have four cash flows of £2,000 each. These are illustrated in the time line below;

Year 0 1 2 3 4 \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

£2000 £2000 £2000 £2000

These cash flows have to be brought back to a present value. One way of valuing them would be to use Table A1.1 and value each cash flow individually. This is time consuming and the annuity method is much quicker. The expression for the annuity valuation is;

PVA = FV \* PVAIFi,n

Where PVA is the present value of the annuity.

The process is the same as before. The future value is the stream of £2,000s, the interest rate is 7% and the time is 4 years. This time we are using Table A1.2. Look up 7% and 4 years to get 3.3872. The values can be inserted into the expression;

PVA = £2,000 \* 3.3872

PVA = £6,774.40

This is much quicker than using Table A1.1 to present value each individual cash flow. To set your mind at ease, in case you are unsure of the calculation, try present valuing them the long way and compare your answers. The values that you find in Table A1.2 are simply the sum of the present values up to that point in time. So for the four year annuity at 7% of 3.3872, this is the one year, two year, three year and four year present value factors from Table A1.1 added together.

This is

|  |  |
| --- | --- |
| Year | Present value interest factor (PVIFi,n) |
| 1 | 0.9346 |
| 2 | 0.8734 |
| 3 | 0.8163 |
| 4 | 0.7629 |
|  | 3.3872 |

If you have a financial calculator it can calculate the present value of annuities. A scientific calculator will not calculate the present value of the annuity quickly.

Example.

A company signs a contract and will collect £120,000 per year for six years, with the first payment at the end of the first year. The appropriate interest rate is 6%. What is the present value of the cash flows?

Solution:

Use the present value of annuity expression and look up Table A1.2, 6 years and 6% for the annuity factor;

PVA = FV \* PVAIFi,n

PVA = £120,000 \* 4.9173

PVA = £590,076

The contract is worth £590,076.

**How do you value an annuity due?**

An annuity due is when the cash flows are at the start of the year, rather than the end of the year. So the tables cannot be used directly. The can be used with a slight adjustment;

Present value of an annuity due:

PVA = FV \* PVAIFi,n \* (1 + i)

All you are doing is multiplying the original annuity factor by (1 + i) to reflect the fact that the cash flows are a year earlier. The present value of the annuity due will be greater than the ordinary annuity because the cash flows have been shifted one period forward and so are not discounted as much as with the ordinary annuity.

The annuity due is valued in a similar manner to the ordinary annuity, but the annuity table (A1.2) will discount the cash flows by one period too many, so we need to multiply by (1 + i) to reflect the earlier timing of the cash flow.

Example.

With the previous example, the company signed a contract to receive £120,000 per annum for six years. Assume that the first payment is today instead of a year’s time. The interest rate is 6%.

What is the present value of the cash flows?

As the cash flows start today (instead of the end of the year) it is an annuity due and we have to use the expression;

PVA = FV \* PVAIFi,n \* (1 + i)

PVA = £120,000 \* 4.9173 \* (1 + 0.06)

PVA = £625,481

If you make a mistake with the timing in an annuity valuation, there can be quite a financial cost to the mistake. Whenever you see annuity type questions always check the timings of the cash flows first, to establish whether you are valuing an ordinary annuity or an annuity due.

**Where would you use the annuity short cut?**

Finding the interest rate

Worked example:

If you ran a business and are used to borrowing money at a certain rate of interest you know what your repayments on a loan would be. Say you were approached by company selling equipment that you need, who offered you a special deal if you bought their machinery.

The equipment would cost £300,000 to buy, but the seller is willing to let you buy it if you pay them £73,167 each year, for five years, starting from the end of the first year. Your normal borrowing interest rate is 8.5%. What interest rate are you paying if you buy the equipment?

Solution

You will start with the present value annuity expression

PVA = FV \* PVAIFi,n

You identify the variables that you have; PVA (this is the £300,000 value of the equipment), FV (this is the annual cash flow you have to pay in the future), and *n* (there are five years cash flows to pay). The missing item is the interest rate, which is part of PVAIFi,n . Rearrange the expression to solve for the missing item;

PVAIFi,n = PVA  
 FV

PVAIFi,5 = £300,000  
 £73,167

PVAIFi,5 = 4.1002

This is the present value annuity factor for the problem. You will find this in Table A1.2. Look up the table and read along row 5 (periods) until you come to 4.1002, then read up for the interest rate. This is 7%. This is lower than your normal borrowing interest rate of 8.5%, so you would go ahead and buy the equipment from the company.

Example

Biffo Enterprises borrows £250,000 from the bank for ten years and arranges to repay the loan in equal instalments of £35,594 at the end of each year. What is the interest rate being charged by the bank?

Solution

Use the expression;

PVA = FV \* PVAIFi,n and adjust to solve for the missing interest rate (PVAIFi,n);

PVAIFi,n = PVA  
 FV

PVAIFi,10 = £250,000  
 £35,594

PVAIFi,10 = 7.0236

Look up Table A1.2, read along the row for ten years, then up for the interest rate when you find 7.0236. The interest rate is 7%.

**Find Cash flow to service annuity**

We have used the present value of annuity expression to find the interest rate that would apply to a loan give the repayments required on the loan. We can also find the annual cash flow required to service a loan given the interest rate and term of the loan.

Worked example.

You want to purchase a piece of machinery and the terms the seller has offered you are; a price of £110,000 including a deposit of £10,000 and the balance to be paid off at an interest rate of 12% over five years, with the first payment at the end of the first year. What would be your annual payments?

Again the present value annuity expression is used;

PVA = FV \* PVAIFi,n

We need to rearrange to solve for the missing cash flow (FV in the expression);

FV = PVA  
 PVAIFi,n

Look up Table A1.2 for the PVAIF0.12,5, which is 3.6048

FV = £100,000  
 3.6048

FV = £27,740.97

A payment of £27,740.97 at the end of each year will service the loan, paying interest and repaying an amount of principal each year, until at the end of year 5 there will be a zero balance. This is known as the loan amortisation and for this loan is shown below;

Loan Amortisation

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Balance | £100,000 |  | Interest rate | 12% |
|  |  |  | Annuity factor | 3.6048 |
|  |  |  | Annuity payment | 27740.97 |
|  |  |  |  |  |
|  | Annuity | Part of the annuity: | | Outstanding balance |
| Year | payment | Interest | Principal | after annuity payment |
| 1 | 27740.97 | £12,000.00 | £15,740.97 | £84,259.03 |
| 2 | 27740.97 | £10,111.08 | £17,629.89 | £66,629.14 |
| 3 | 27740.97 | £7,995.50 | £19,745.48 | £46,883.66 |
| 4 | 27740.97 | £5,626.04 | £22,114.93 | £24,768.73 |
| 5 | 27740.97 | £2,972.25 | £24,768.73 | £0.00 |

This shows that in the first year a payment of £27,740.97 will be made up of £12,000 interest (£100,000 at 12%) and the remainder of £15,740.97 being the repayment of principal. So at the end of the first year, you no longer owe £100,000, the balance has been reduced by £15,740.97 and the outstanding balance is £84,259.03 and that is the amount that you will pay interest on in the second year. So in the second year the amount of interest paid falls and the amount of principal repaid rises.

This is exactly the same principle behind your mortgage and your mortgage repayment schedule would look similar to this, but on a monthly basis (so you would work out a monthly annuity factor, eg for a twenty year mortgage with monthly repayments, you would work out a 240 period annuity factor, which you would divide into your mortgage amount).

**Finding the cash flow in an annuity due example**

Taking the above equipment purchase, assume there is no deposit, and the entire £110,000 purchase price had to be financed with a five year, 12% bank loan, but here the first payment is today rather than the end of the year. What are the annual payments this time?

Solution;

This is an annuity due problem, where the cash flows have been moved to the start of the year. We still use the present value annuity expression, but we need to adjust for the fact that the cash flows are paid a year earlier;

PVA = FV \* PVAIFi,n \* (1 + i)

We need to solve for the missing item, which is the cash flow (FV);

FV = PVA  
 PVAIFi,n \* (1 + i)

FV = £110,000  
 3.6048 \* 1.12

FV = £110,000  
 4.037

FV = £27,245.42

You can see that the loan repayments are lower than in the original case (even though the borrowing is higher), this is because you are foregoing a year’s cash flow with the annuity due example.

**Solving for the number of periods**

We have seen that with the present value annuity expression, if you have three of the variables in the expression it is straight-forward to solve for the missing item. The expression has just to be rearranged to solve for the missing variable. Finding the number of periods that applies to an annuity is no different.

Example

A new business is wishing to borrow some cash to buy some servers for their operation. Buying and installing the servers will cost £60,000. This is the largest amount they have ever borrowed and would like to repay the debt as quickly as possible. They have made cash flow projections and feel that they would have £18,928 available at the end of each year to pay off the debt. If the borrowing rate was 10% , how many years would they be borrowing for?

Solution

Use the present value annuity expression;

PVA = FV \* PVAIFi,n

This time the missing item is time, which is contained in the PVAIFi,n, so solve for that missing item;

PVAIF0.10,n = PVA  
 FV

PVAIF0.10,n = £60,000  
 £18,928

PVAIF0.10,n = 3.1699

Look up Table A1.2 and read down the 10% column until you come to 3.1699 and read along for the number of years. They will pay off the debt in four years. This particular method can be used for managing personal finances, for example, when arranging a new mortgage, you may feel that you are able to pay a certain level of cash flows each month out of your salary. This can be inputted into this expression to work out a time period to pay off your mortgage, which might be quicker than what the bank is offering. Meaning you are debt free quicker.

**Future Value of annuities**

Mostly with annuities we are calculating the present value, but there are times when the future value of an annuity needs to be calculated. This may be when you are saving for something, or when a company needs to redeem a bond and needs to put aside cash each year to repay the bond principal.

The format for calculating future value of annuities is very similar to that of the present value of annuities. This can be seen with the following example. You want to buy a holiday home in France and need to save up for a deposit. You need a deposit of £35,000 by the end of four years, and you can save £7,500 per annum. Your savings will earn 8% interest during this time. Will you have enough cash in the bank to pay the deposit by the end of four years?

Solution

The framework for solving future value of annuity problems is;

FVA = PV \* FVAIFi,n

It is important to show the timeline for a future value of annuity problem here.

Year 0 1 2 3 4 \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

£7500 £7500 £7500 £7500

With an ordinary future value of annuity, the first cash flow saved is at the end of the first year and for this four year future value annuity, the final cash flow is paid as the annuity matures, so the final payment attracts no interest. The annuity will calculate the compounding effect of the interest over the years for the series of payments.

FVA = £7,500 \* FVAIF0.08,4

Look up Table A1.4, 8% and 4 years to obtain 4.5061.

FVA = £7,500 \* 4.5061

FVA = £33,795.75

Saving £7,500 each year starting at the end of the first year will not reach the target of £35,000. But what if the saving started today, rather than the end of the first year? This would be a future value annuity due.

**Future value annuity due**

This works in exactly the same way as with the present value annuity due. The annuity factor is multiplied by (1 + i) to represent the extra year of interest earned on the payments. Using the example from above, we have

FVA = PV \* FVAIFi,n \* (1 + i)

FVA = £7,500 \* 4.5061 \* (1.08)

FVA = £7,500 \* 4.8666

FVA = £36,499.50

If you start saving today, rather than the end of the year, then the deposit target of £35,000 will be reached and passed.

With annuities it is important to establish the timing of the cash flows when you commence the calculation.

**Finding the interest rate with future value of annuities**

There can be times where a target cash flow and the amount of annual savings is known, and the number of years needed to reach the target, the missing item is the interest rate needed to get there. With the financial tables this becomes an easy calculation, using the future value annuity framework;

FVA = PV \* FVAIFi,n

Solve for the missing interest rate;

FVAIFi,n = FVA  
 PV

Example

A company needs to repay a bond at the end of six years. The bond liability is £200m and the company can put aside £30m at the end of each year. What rate of interest is needed on their bank savings to meet the £200m target at the end of six years?

Solution

FVAIFi,n = £200m  
 £30m

FVAIFi,n = 6.6667

Look up Table A1.4, read along row six until you come to 6.6667. At 4% the FVAIFi,n is 6.6330, which is almost the required future value factor, but not quite. We can say that the interest rate needed is slightly over 4%.

What if the company could only pay in £25m at the end of each year? The future value factor would be 8.0000 and looking along row six for 8.0000, we find that it lies in-between 11% and 12%. The interest rate needed would be just under 11.5% per year.

**Solve for missing cash flow**

By now the format for these calculations should be clear. The future value of annuity expression is used;

FVA = PV \* FVAIFi,n

The annual cash flow (PV) is missing, so solve for that missing variable;

PV = FVA  
 FVAIFi,n

Example

A company wants to borrow £400,000 for five years. It wants to know at the outset how much it will have to put aside at the end of each year, in a bank, if it wants to be able to repay the £400,000 at the end of the five year period. They can earn 7% on their savings. How much do they need to save at the end of each year?

Solution

PV = £400,000  
 5.7507

PV = £69,556.75

If the company saves £69,556.75 at the end of each year they will reach the target of £400,000 by the end of year 5 and they can then pay off the loan with the proceeds.

**Special features of annuities**

Annuity tables can be used to value level cash flows that occur in the future for short periods.

For example, a company has entered a contract which will see them receive £300,000 per year for five years, with the first payment being received at the end of the fourth year. The interest rate is 8%, what it the present value of this stream of cash flows?

You need to draw a timeline of the problem, showing when the cash flows take place and what happens when you present value the annuity using the tables.

Timeline

Year 0 1 2 3 4 5 6 7 8 9 \_\_ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

300 300 300 300 300

PV3

PVA = FV \*PVAIF0.08,5

PVA = £300,000 \* 3.9927

PVA = £1,197,810

When you present value an annuity it brings it back to a value one year earlier than the first payment. So when we use Table A1.2 to present value this stream of level cash flows, the present value is not at time 0, it is at the end of year 3. So we have the present value at the end of year 3, we need the present value at time zero, ie, now. What do you do next? The next stage is just a simply present value of a single cash flow (PVIFi,n). Look up 8% and 3 years in Table A1.1, obtaining a present value interest factor of 0.7938 and multiply the PVA by this to obtain the present value.

PV = FV \* PVIF0.08,3

PV = £1,197,810 \* 0.7938

PV = £950,822

**Perpetuities**

Perpetuities are cash flows that go on forever. It is an annuity that has an infinite lifespan. Where do these occur in finance? Dividends on preference shares are perpetuities. When a company issues equity onto the stock market it is effectively doing it forever. The shares are not going to be redeemed after twenty years and the company wound up. When a company comes to the market with its shares it is issuing a perpetuity. The preference shares are a normal perpetuity and the ordinary shares (or common stock) are a growing perpetuity (because they usually payout higher amounts to shareholders each year). There are some bonds that are perpetuities, which means they will never be redeemed. The UK Government bond market has a perpetuity known as ‘War Loan’, issued in 1917 to help fund the First World War and had a coupon of 5% (later cut to 3.5% in 1932). The bond has never been redeemed.

**How to value perpetuities**

The value of a perpetuity is simply the cash flow provided by the perpetuity divided by the required rate of return. If we take the War Loan 3.5% bond from above and if we assume the required rate of return is 4%, then the present value of the perpetuity (PVP), ie. the bond)is;

PVP = FV  
 r

where r is the required rate of return.

PVP = 3.50  
 0.04

PVP = £87.50

If the required return was 3%, then the bond would be valued at £116.67.

In the perpetuity valuation the FV in the calculation is the next period’s cash flow, so in this case, next year’s coupon payment.

**Growing perpetuities**

A perpetuity is just a constant cash flow into the future. Sometimes cash flows will grow into the future. A company’s dividend stream is an example of a cash flow that can grow into the future. The value of a company can be based on the dividends that they pay out. When a company comes to the stock market, it does so for an indefinite period, so companies on the stock market can be valued as growing perpetuities.

The formula for a growing perpetuity is;

V =

Where, CF = cash flow, i = interest rate and g = growth rate of cash flows.

With a share valuation, the CF become dividends (D) and i becomes the required return on equity (re) and g is the growth rate of dividends.

The dividend in the calculation is the next dividend, because the value of equity is the present value of all future dividends. Past dividends have been paid and have been taken out of the share price.

The formula for a share valuation is;

P0 =

Example;

If a company has just paid a dividend of 20p and the required return on equity is 14% and the growth rate of dividends is 4%, what is the current share price?

Solution;

The dividend given in the example is the current dividend, D0, which has been paid, so it is no longer part of the share price. The next dividend will be the current dividend multiplied by one plus the dividend growth rate, ie;

D1 = D0 \* (1 + g)

D1 = 20 \* 1.04 = 20.8p

Then we can use the growing perpetuity formula to calculate the share price;

P0 =

P0 =

P0 = 208p

We will come back to this formula in Module 2 when we look more closely at share valuation.

**In Summary**

**Which table do I use?**

Answer two questions;

When do I need the money?

Is it a single cash flow or a multiple cash flow?

Your answers will point you to the correct table. Do you need the money now (Table A1.1 or A1.2) or in the future (Table A1.3 or A1.4).

Is it a single cash flow (Table A1.1 or A1.3) or a multiple cash flow (Table A1.2 or A1.4).

The financial tables are a very powerful tool in helping you with your financial decision making. The basic expression PV = FV \* PVIFi,n can be adapted to solve for missing cash flows, interest rates, number of periods for different present and future value situations covering single cash flows and multiple equal cash flows.

The annuity framework is very useful in using with equal cash flows over time. Mixed unequal cash flows have to be valued individually.